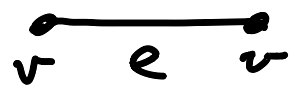
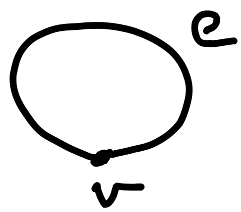
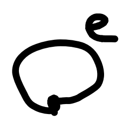
多面体グラフとオイラー標数

頂点(vertex, v) 

辺(edge, e) 



rules

(a) 辺は  or  と

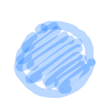
位相同型　　　　loop

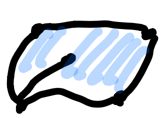
(b) $e_1 \ni P$, $e_2 \ni P \Rightarrow P$ は頂点.

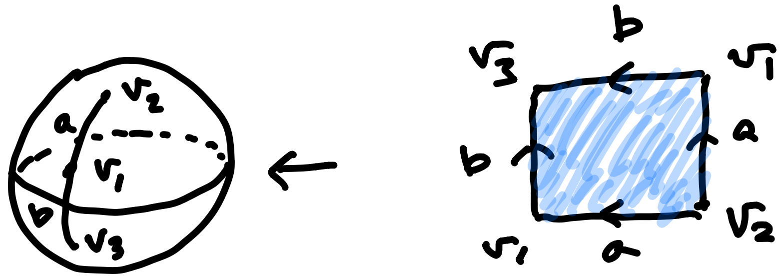
すべての頂点と辺とをあわせた図形を

<u>グラフ</u>という.

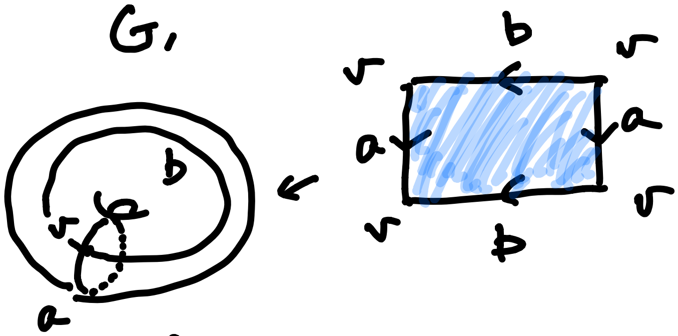
・open disk  と位相同型の

領域を面(face)という 

 $S \Rightarrow S - \mathrm{Bd}\,S$: 



G_1

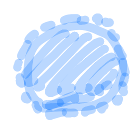


G_2

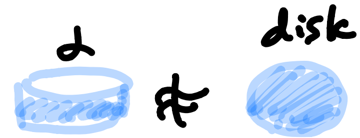
すべての領域がfaceである閉曲面Sの連結グラフを多面体グラフという

$\mathrm{Int}\, G_1 \approx \mathrm{Int}\, G_2$

$\approx$ open disk



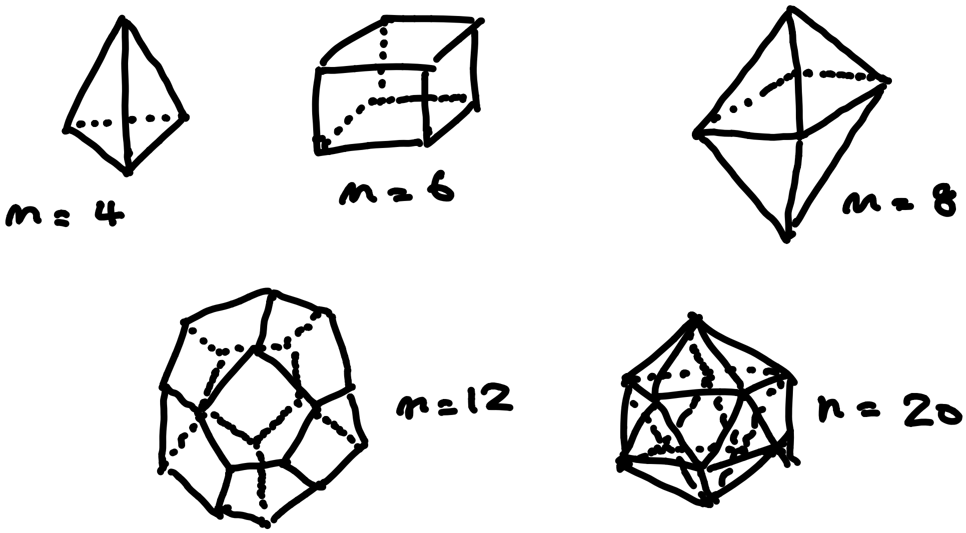
多面体グラフでない



$$\chi(S) = \text{Sの頂点の数} - \text{辺の数} + \text{面の数}$$

$$\chi(S, G_1) = \chi(S, G_2)$$ グラフの選び方によらない

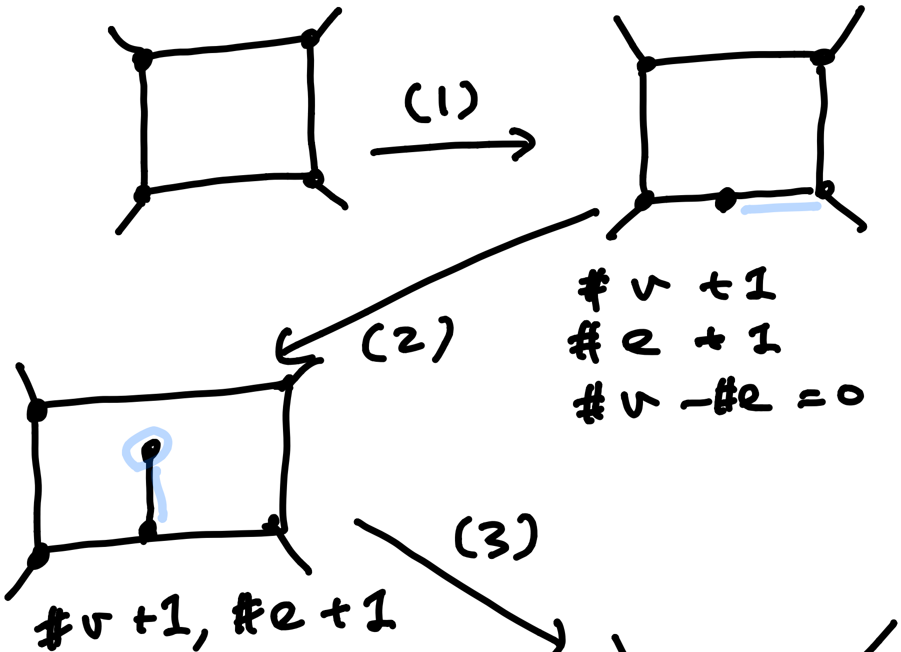
正n面体

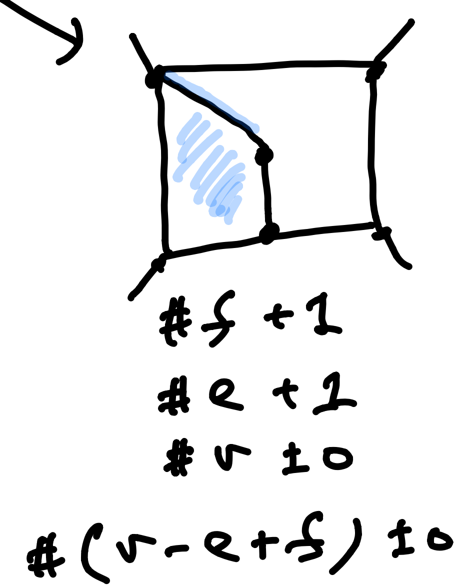


n	#v	#e	#f	#(v−e+f)
4	4	6	4	2
6	8	12	6	2
8	6	12	8	2
12	20	30	12	2
20	12	30	20	2

$\chi = \#v - \#e - \#f$

閉曲面Sの任意の2つの多面体グラフ $G_1, G_2 \Rightarrow \chi(G_1) = \chi(G_2)$





定理

任意の2つの閉曲面 S, S'

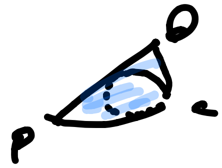
$S \approx S'$

$\Rightarrow \chi(S) = \chi(S')$

$\exists f : S \to S'$ 同相

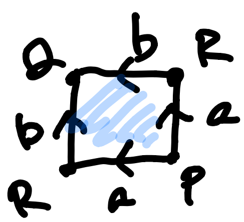
$\chi(S) = \chi(S, G) = \chi(S', G') = \chi(S')$

$\chi(S^2)$ $\quad cc^{-1} = 1$



G_1

$$\left.\begin{array}{l} \# v = 2 \\ \# e = 1 \\ \# f = 1 \end{array}\right\} \Rightarrow \chi(S^2) = 2 - 1 + 1 = 2$$



G_2

$\# v = 3 \ (P, Q, R)$

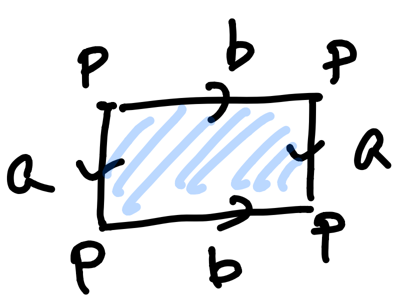
$\# e = 2 \ (a, b)$

$\# f = 1$

$3 - 2 + 1 = 2$

$$\chi(S^2, G_1) = \chi(S^2, G_2) = \chi(S^2)$$

$\mathbb{T}^2$:



$\# v = 1$

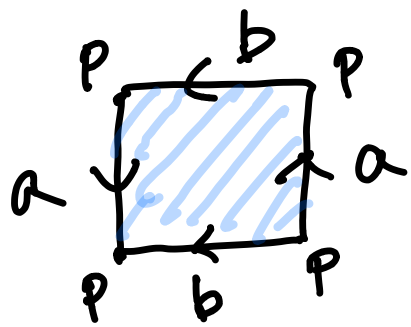
$\# e = 2$

$\# f = 1$

$$\chi(T^2) = 1 - 2 + 1 = 0$$

定理の
対偶より。 $\mathbb{T}^2 \not\approx \mathbb{S}^2$

$\mathbb{K}$



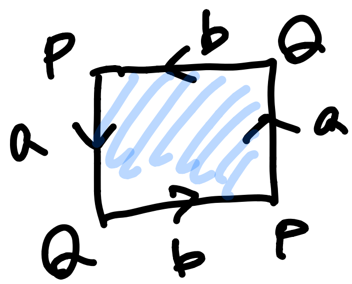
$\# v = 1$

$\# e = 2$

$\# f = 1$

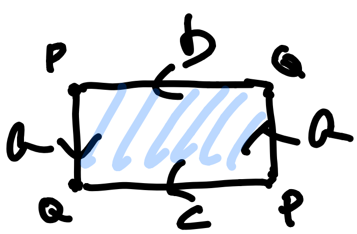
$$\chi(\mathbb{K}) = 1 - 2 + 1 = 0$$

$\mathbb{P}^2$



$$\chi(\mathbb{P}^2) = 2 - 2 + 1 = 1$$

$\mathbb{M}$



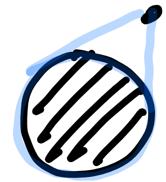
$$\chi(\mathbb{M}) = 2 - 2 + 1 = 1$$

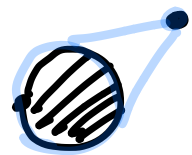
$$\chi(\mathbb{K}) \neq \chi(\mathbb{P}^2)$$

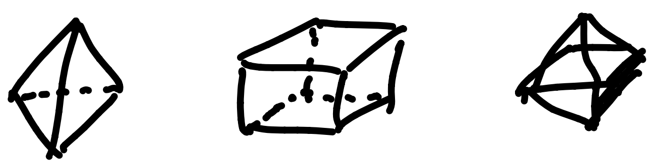
$$\Rightarrow \quad \mathbb{K} \not\approx \mathbb{P}^2$$

S^2 のグラフ（球面におけるグラフ）

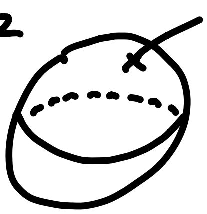
	#v	#e	#f	χ
G_0	1	0	1	2
$G(k,2)$	2	k	k	2
$G(2,k)$	k	k	2	2
正4面体	4	6	4	2
〃6〃	8	12	6	2
〃8〃	6	12	8	2
〃12〃	20	30	12	2
〃20〃	12	30	20	2



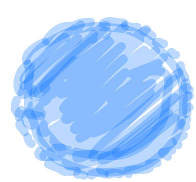




S^2 v



$S^2 \setminus \{v\} \approx$ open disk



$\chi(S^2)=2$ ギリシャ人知っていた